QUANTUM GRAVITY VIA NONCOMMUTATIVE GEOMETRY

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1. Background

Much of twentieth-century physics has been concerned with the parallel development of two seemingly fundamental, yet inexplicably separate, theories of nature. The first is Einstein's general relativity, a large-scale theory of space, time, and gravitation. The second is quantum theory, culminating in the Standard Model of particle physics, which explains essentially everything non-gravitational in the universe.

The fact that nature is a single entity means that we should be able to combine these two theories into a single, self-consistent mathematical framework which describes the entire natural world—certainly a worthwhile goal. However, the most straightforward ways to do so fail painfully, producing mathematical singularities such as predictions of infinite self-energy of the newly-quantized gravitational field. In light of this, several research programs arose in the late twentieth century aiming to apply more specialized mathematical techniques to the problem of this unification, some of the most prominent including string theory and Ashketar's loop quantum gravity [1].

At the same time, not all is well in the realm of quantum theory. The Standard Model is only one of many quantum field theories, based on several arbitrary choices—such as that of the gauge group $U(1) \times SU(2) \times SU(3)$ or the particle representations—and composed of a mathematical hodgepodge of techniques under names such as the Higgs mechanism, V–A symmetry, and spontaneous symmetry breaking [2]. Over the last thirty years, a promising new set of mathematical techniques for explaining the Standard Model has arisen under the heading of noncommutative geometry.

The essence of noncommutative geometry is to exploit the categorical equivalences that exist between algebraic and geometric structures, seen for example in the celebrated Gelfand–Neumark theorem regarding the correspondence between the algebraic objects known as commutative C^*-algebras, and the geometric objects known as topological spaces [3]. By starting with *noncommutative* algebraic structures, where $xy \neq yx$, we retrieve a rich geometric structure including modified notions of the concepts such as points, distances, and differentials. This mathematical framework is then perfect for quantum theory, one of the primary features of which is the noncommutativity of observables (summed up pithly in the famous Heisenberg uncertainty principle).

Work by Chamseddine, Connes, Marcolli, and others has shown how the Standard Model, coupled minimally to gravity, emerges naturally as a purely gravitational theory over a noncommutative spacetime in the form $M \times F$ [4]. Here M is our usual four-dimensional spacetime manifold, and F is "one of the simplest and most natural finite noncommutative geometries of the right dimension (6 modulo 8) to

solve the fermion doubling problem" [2]. Parameters that are seemingly arbitrary in the usual formulation become geometrical invariants in this new Noncommutative Standard Model. The techniques of noncommutative geometry are crucial in this construction, as considerations of the diffeomorphism group would make it impossible for a gravitational theory to give rise to the complicated structure of the Standard Model when applied to a commutative space. Also impressive is the way in which these techniques constrain the theory: the Noncommutative Standard Model makes testable predictions about such things as the range of the Higgs sector.

2. Research Program

Yet for all its successes, the Noncommutative Standard Model still does not solve the question we began with: namely, how can we formulate a quantum theory of gravity?

At first glance, this can be confusing: after all, the Noncommutative Standard Model is a theory of gravity on a noncommutative space, and gives rise to the Standard Model—usually thought of as an essentially quantum model. It turns out that, despite the seeming integration of the two concepts, the Noncommutative Standard Model still has the essential dichotomy between classical gravity and quantum theory buried within its formulation. One proceeds with purely gravitational techniques in the model's construction, and then expands the resulting action into a gravitational sector involving only spacetime metric fluctuations and a matter sector involving all the particle fields of the usual Standard Model. One then applies quantum field-theoretic techniques only to this latter sector, retrieving the usual procedures and results [5]. Thus, spacetime and the gravitational force are themselves never quantized, and attempting to do so gives rise to the usual problems.

We propose to investigate a different approach to quantum gravity than that produced directly by the Noncommutative Standard Model. Our research program is guided by considerations as to the naturalness of noncommutative techniques for creating an essentially quantum spacetime. Just as quantum theory replaces classical phase-space coordinates (i.e. position and momentum) by noncommuting operators, it is possible to replace the classical spacetime coordinates of general relativity with generators of a noncommuting algebra. This gives rise to a picture of spacetime, not composed of problematic infinitesimal points, but of smeared-out cells at a fundamental length scale. The techniques of noncommutative geometry then allow us to construct a spacetime invariant under the appropriate symmetry groups, and seem likely to remove the usual mathematical singularities (thus fulfilling the old idea of Pauli that the gravitational field will somehow regularize the divergences of field theories).

Such a prescription is actually rather broad, with many possible promising approaches (see e.g. [5] and references contained therein). Some tentative work has been done on applying quantization techniques to noncommutative spaces, especially with regards to the Noncommutative Standard Model [6, 7]; the results indicate that the natural setting may be the noncommutative space arising from the algebra of holonomy loops on certain functional spaces of spin-connections. This is especially interesting given the similarity of this framework to the spin-network formalism used in loop quantum gravity, and we suspect that noncommutative geometry may be able to contribute substantially to the loop quantum gravity program. It would also be

interesting to see how the formulation of the Standard Model via the loop-theoretic ideas of [8] fits with noncommutative ideas (if at all).

3. Specifics

In light of the above, we request funding for a 2009 Summer Undergraduate Research Fellowship investigating applications of noncommutative techniques to quantum gravity, with Caltech professor Matilde Marcolli as a mentor and myself, Domenic Denicola as the student researcher. Professor Marcolli's field of research is the Noncommutative Standard Model and extensions thereof; during the Fall 2008 term, I took a class taught by her titled "The Geometry and Arithmetic of Quantum Fields" in which the Noncommutative Standard Model, and associated mathematical techniques, were explained in great depth. Our broad goal is to produce publishable results showing novel and definitive contributions of the promising mathematical techniques in noncommutative geometry to the problem of quantum gravity. This would be highly significant for the physics community, for as explained in the introduction, the problem of quantum gravity is one of the most interesting, most fundamental, and most technically difficult facing the field today.

Although the research is firmly couched in the world of abstract mathematics and theoretical physics, and thus it is difficult to plan exactly how it will go, we do have some concrete approaches which we plan to follow over the SURF period. Investigation of existing work has already begun, as can be seen from the References section below; it will continue throughout the remainder of the school year, and the first week or two of the research period will consist of consolidating this knowledge and using it to guide research going forward. Soon after this step, we will find a specific area within our broad goal that seems likely to give concrete results, whether this be the application of our techniques to unexplored areas or the tackling of a specific problem within existing research.

An example of the former would be to apply noncommutative techniques to Julian Barbour's Machian Quantum Gravity program [9], which I have recently collaborated with Dr. Barbour on in person. An example of the latter would be to pursue the aforementioned integration of noncommutative geometry and loop quantum gravity, which as can be seen from the associated references has tentatively begun but has some technical difficulties. Work on the latter would probably include travel to the Perimeter Institute for Theoretical Physics in Canada, which is a center of loop quantum gravity research, to work with some colleagues I have been in contact with there. To be clear, both of these are ready-to-go research ideas that have been discussed and investigated in some depth, and which I could tackle competently: we are not applying for the Fellowship on the vague hope of finding something suitable over the course of the research period itself. But we feel that deciding between them, and also between other possibilities that are more vague at this time, will require dedicated research time during the summer.

Once a specific area is decided on, the usual process of theoretical research begins, as we explore various approaches, solving problems and acquiring the necessary background as things come up. Frequent meetings between myself and Professor Marcolli will take place, as her experience and insight will be invaluable in guiding my progress and helping me find appropriate resources. By the end of the summer, assuming a solution, one or more papers should be in preparation for publication in appropriate journals.

References

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